

Ice Technology for Polar Operations

Proceedings of the Second International Conference on Ice
Technology, held at Downing College, Cambridge University,
UK, during 18-20 September 1990

Edited by: T.K.S. Murthy
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Computational Mechanics Publications
Southampton Boston

Wave Energy and Floe Collisions in Marginal Ice Zones

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ABSTRACT

The mechanical interaction between discrete ice floes in a Marginal Ice Zone is addressed in terms of its effect on wave energy attenuation and floe destruction. Collision characteristics are estimated from a simple hydrodynamic model and are coupled with a mechanical model of ice impact. Wave induced floe collisions displace brash ice surrounding the floes and cause large local stresses which damage, pulverize and extrude the ice parallel to the contact face. These processes are shown to result in essentially inelastic floe collisions. Thin-layer fluid and constant stress models for contact pressures are used to give upper bounds on floe size reduction and brash ice production which is the residue of the crushing process. The energy lost in floe collisions does not play a large role in wave attenuation.

INTRODUCTION

Some Marginal Ice Zones (MIZ's) exhibit well defined changes from the interior pack through a transition zone to the edge zone [1,2]. Others, such as the Labrador ice Margin, are dispersed by winds and currents leaving a much more open ice cover and ill defined edge. Here hundreds of kilometres of ocean surface can be covered by bands and patches of ice intermixed with bands and patches of open water. Ice thicknesses can also vary considerably from one marginal zone to another, and to a lesser extent within MIZ's.

However, despite these macroscopic morphological differences, the fundamental mechanics of ice destruction are essentially the same. The large diameter floes characteristic of the interior pack are broken into smaller pieces by long period propagating swell, which causes flexural failure as the floes bend over the waves and as the floes collide. Once the floes have been reduced in size to some critical diameter (≈ 10 to 20 m) they act more like rigid bodies and bending strains are insufficient to induce failure despite increasing wave amplitudes and frequencies as the floes near the open water. In this outer region of the MIZ there are two dominant destruction processes: melting and mechanical dissolution during floe collisions. The low melt rates which have been observed in many MIZ's ([3],

for example) suggest that the latter process predominates. This hypothesis is supported by qualitative observations of floe size reductions in the Labrador Ice Margin.

The frequency of these collisions and the energy associated with them is a function of incident wave amplitude and period, and the sizes and shapes of the floes themselves. If the floes vary significantly in size and shape they may respond differently to a given wave field, causing frequent collisions. If the floes are fairly uniform in size, which is generally the case in any given section of a MIZ, their dynamic response to a wave will be similar. They will, however, be slightly out of phase because of the differences in their average position on the wave. The collisions which result from this phase difference are described below.

Although wave propagation into MIZ's has been studied extensively, the effect of energy loss due to floe collisions has not been rigorously investigated. Several field experiments (the MIZEX and LIMEX programmes for example) have produced direct measurements of wave energy attenuation and estimates of the energy attenuation coefficient α [4], but these measurements provide little insight into the nature of the attenuation process. In this paper we develop a theoretical model of floe collisions and ice deformation, and associated wave energy losses in the MIZ.

WAVE INDUCED FLOE COLLISIONS

For a simple, harmonic, deep water, ocean wave of amplitude a , angular frequency ω , and wave number k , the surface elevation (y_n) at any time t and horizontal distance x_n is

$$y_n = a \sin(kx_n - \omega t) \quad (1)$$

where the subscript n indicates mean horizontal position on the wave. The surface slope at time t is therefore

$$\frac{dy_n}{dt} = -a\omega \cos(kx_n - \omega t). \quad (2)$$

The horizontal (u) and vertical (w) components of the water particle velocities are

$$u_n = a\omega \exp(kz) \cos(kx_n - \omega t) \quad (3)$$

and

$$w_n = a\omega \exp(kz) \sin(kx_n - \omega t), \quad (4)$$

where z is the depth below the mean surface. At the surface $z = 0$, so that (3) and (4) reduce to

$$u_n = a\omega \cos(kx_n - \omega t) \quad (5)$$

and

$$w_n = a\omega \sin(kx_n - \omega t). \quad (6)$$

The coordinates of the fluid particle measured from its mean position are then

$$\xi_{n1} = -a \sin(kx_n - \omega t) \quad (7)$$

and

$$\xi_{n2} = +a \cos(kx_n - \omega t). \quad (8)$$

If we assume for the moment that effects of added mass and non-unity response amplitude operators (RAO's or "transfer functions") can be ignored, then these formulae can also be used to describe the movements of two ice floes separated by Δx . The

positions of the floe centres at t are described by equations (7) and (8), offset in the vertical coordinate (ξ_{n3}) by

$$\frac{\rho_i h}{\rho_w} - \frac{h}{2}, \quad (9)$$

(where ρ_i and ρ_w are the densities of ice and water and h is the ice thickness) corresponding to the submergence of the ice due to the density difference between it and the water. Equation (2) is also valid for two floes if we assume that the slopes are tangents to the water surface at the floe centres.

Figure 1 is a schematic representation of the motion of two ice floes one metre thick and 10 m in diameter, on a 1 m amplitude sinusoidal wave of frequency 0.1 Hz. The floes, which are separated by an average distance of 10 m, collide 2.9 s into the cycle. In the diagram, they are shown to overlap for a period of approximately 5 s. In reality this would not occur; however, it is during this contact interval that damage may take place. The straight lines emanating from the geometric centres of the floes are velocity vectors, while the small crosses indicate the centres of rotation. The dashed line indicates the water surface. Because the orbital motion is circular (it is a deep water wave) the magnitudes of the velocity vectors are constant. The collision velocities are therefore controlled by the vector directions at the instant of impact.

Figure 2 shows calculated collision velocities for floes in a 0.1 Hz wave over a range of wave amplitudes and floe diameters. Here the term 'collision velocity' refers to the differential x (horizontal) component of the velocity vectors. We have ignored the vertical component of the differential velocity at impact as it is always less than 10% of the horizontal component over the range of conditions which have been modelled. From Figure 2, collision velocity appears to vary linearly with wave amplitude and floe diameter. This is confirmed by looking at the difference in horizontal velocity from equation (5) for two points separated by one floe diameter, e.g. $2r$,

$$u_1 - u_2 = a\omega[\cos(-\omega t) - \cos(2kr - \omega t)]. \quad (10)$$

The maximum velocity difference in one wave cycle is

$$v = 2a\omega \sin(kr), \quad (11)$$

which for small kr is

$$v = 2\omega kr. \quad (12)$$

The result (12) is a very close approximation to the relation shown in Figure 2.

In all cases it is assumed that the floe centres are, on average, separated by a distance equal to their diameters. The separation distance is rather important to the collision velocity calculations. Maximum collision velocities occur when the floe centres are placed one diameter apart, and there are critical separation distances at which the floes either never meet or never separate. The nature of this relationship is illustrated in Figure 3 for the standard 10 m diameter floes in a 1 m amplitude wave of period 10 s.

The model described above is a gross simplification of real floes in a real wave field. Particularly conspicuous by their absence are the effects of non-unity transfer functions of wave induced floe motion, and of added mass. In the case of the transfer functions (RAO's), we have not performed the rigorous calculations necessary to determine the floes' response, but have utilized the results of other studies. Masson and LeBlond [5] for example, computed surge, heave and pitch RAO's for a range of wave conditions and floe geometries and sizes. Their results indicate that for the floes modelled in this study the response amplitude operators are very close to unity for all three motions. Having assumed that the floes closely follow the sea surface, it follows that their added mass due to wave induced motions is negligible.

MECHANICAL PROCESSES DURING IMPACT

Attempts have been made to model pack ice behaviour in marginal ice zones by considering the elastic collisions between floes [6,7]. Visual observations of floe collisions in the Labrador marginal ice zone show that this is not so: floes collide then drift slowly apart under wave forces. Particularly following periods where significant ocean swell has penetrated into the pack ice, a large amount of crushed ice is formed. This may be in the form of brash ice that can refreeze between floes or as distinct levees that ring each floe [8]. The presence of the crushed material indicates that a substantial portion of the collision energy has been dissipated. Martin and Becker [9] measured the acceleration of floes in the Greenland Sea marginal ice zone and documented collision events. The floes ranged from 1 m to 6 m in thickness and areal dimensions were in the order of 100 m square. Some collisions were isolated and, when repeated collisions did occur, they generally occurred at the same point in the wave cycle. This would indicate that the collisions were due to wave forces rather than elastic rebound. For massive floes of several km in extent impacting Hans Island, elastic rebound was not a consideration; the floes decelerated gradually as crushing, flexure, splitting and flaking occurred at the contact face [10]. Clearly there is little evidence at any scale for the elastic restitution of floes to play an active role in marginal ice zone dynamics.

Likhomanov and Kheisin [11] dropped steel balls (156 kg and 300 kg) on freshwater ice and noted details of the collision. Restitution coefficients were found to decrease with increasing ice temperature and with increasing impact energy, and varied between 0.05 and 0.3 for velocities of 1 ms^{-1} to 3.5 ms^{-1} . The energy of two 10 m diameter floes colliding at 0.05 ms^{-1} is equivalent to that of a 300 kg mass dropped at 1 ms^{-1} . This floe collision speed is certainly within the range calculated above for wave-induced floe motions. As ice temperatures approached the melting point, elastic restitution was minimal. Restitution for sea ice is also expected to be less than for freshwater ice. Following the above dropped-ball experiments, sections were taken through the ice and the damage was documented by Kheisin and Cherepanov [12]. There was a distinct layer of pulverised ice adjacent to the contact face; there was microcracking next to this layer and macrocracks penetrated the surrounding ice. Since the above experiments were performed, evidence of extrusion and microcracking has been observed in many ice-ship and ice-structure interactions.

While the creep properties of intact ice do not play a significant role during rapid loading, they are considerably enhanced by the presence of microcracks. Crack-enhanced creep has been documented by McKenna [13] for the compression of sea ice at -2°C . Whether the ice at the interface in inter-floe collisions will flow as a fluid or be damaged as a solid depends on the nature of the contact and the state of the ice. The damage process is still being investigated and it is perhaps premature to extrapolate these results to marginal ice zone floe collisions. Simple solid and fluid relations that retain the above attributes are given in the next section.

MODELLING THE COLLISION PROCESS

Based on the above analysis, floe collisions are assumed to be essentially inelastic and motion in the horizontal plane only needs to be considered. The present discussion is restricted to regular circular floes of constant thickness. Particularly in the early development of marginal zones, floes are large, irregularly shaped and eccentric collisions are more probable. Decay through wave action and melting reduces the floes in the Labrador marginal zone to more circular shapes with diameters decreasing with ice thickness. Representative 10 m diameter floes of 1 m thickness are considered.

Consider the non-eccentric collision of two floes of equal mass m and velocities $v_0 + v$ and v_0 shown in Figure 4. If the collision is inelastic, both will assume the same final velocity which is the arithmetic average of the initial velocities. The energy lost in the collision is

$$E = \frac{1}{4}mv^2 \quad (13)$$

and hence is only a function of the relative velocity of the two floes. For a circular floe of constant thickness, the mass is

$$m = \rho_i h \pi r^2, \quad (14)$$

where ρ_i is the ice density, h is the ice thickness and r is the floe radius. For realistic floe geometries in deep water, the hydrodynamic force or added mass coefficient is typically 1 to 4% of the actual mass of a floe [14] and can be ignored in this case. Equating the energy loss to the work expended at the contact interface, one obtains

$$\frac{1}{4}\rho_i h \pi (rv)^2 = \int_0^{s_0} F ds. \quad (15)$$

Here s_0 represents the total penetration distance (see Figure 4) and F is the time varying contact force. Contact geometry is established by considering the penetration of two right circular cylinders. The contact area is rectangular with height given by the ice thickness h and width

$$b = (4rs - s^2)^{1/2}, \quad (16)$$

which depends on the penetration distance s and the radius of the floes r . The energy relation can be rewritten

$$\frac{1}{4}\rho_i h \pi (rv)^2 = \int_0^{s_0} \bar{\sigma} (4rs - s^2)^{1/2} ds, \quad (17)$$

where $\bar{\sigma}$ is the average contact stress. The total volume reduction of the two floes due to the collision is

$$V = \frac{4}{3} h r^{1/2} s_0^{3/2}, \quad (18)$$

as long as the total penetration s_0 is small relative to the floe radius.

While the contact area is simply geometric, the complex factors discussed earlier influence the stress. In general, the interaction is a dynamic process with an alternation between damage and clearing of the pulverised ice. Where substantial melting has occurred or a layer of crushed ice exists at the interface, the dynamics are not significant and the thin-layer fluid model outlined by Jordaan and Timco [15] is appropriate. Here the layer thickness is fixed and the contact stress is the result of extrusion of the crushed ice between the intact ice of each floe. Once penetration has begun, the average pressure is

$$\bar{\sigma} = \frac{\eta v'}{2(s_1 - s)^2} \frac{h^3 b^2}{h^3 + b^3}, \quad (19)$$

in which η is the ice fluid viscosity, v' is the instantaneous relative velocity between the two floes, h is the floe thickness, b is the instantaneous contact width and s_1 is the total fluid layer thickness. This solution is for a uniform thin layer and assumes no flow at the boundary between intact and crushed ice. The relation is useful for estimating penetration distance as long as the impact velocity is not large enough to produce stresses which would result in further damage to the ice and therefore increase the fluid layer thickness. Equation (17) was solved numerically with the average stress defined by (19) to yield the penetration distance s_0 as a function of the initial relative velocity v of the floes and the fluid layer thickness s_1 . The ice fluid viscosity η is the only parameter in the model.

Jordaan and Timco [13] estimated the viscosity of crushed ice to be approximately 0.10 MPa·s at -10°C . For warmer ice in the marginal zone, the viscosity is expected to be higher. Crushed ice volumes were calculated from equation (18) for a range of viscosities and relative floe velocities; these are summarised in Figure 5. A fluid layer thickness of 5 cm was assumed.

While ice behaviour is rate dependent in many situations, the presence of fracture and microfracture can reduce this effect. An alternate way of addressing the interaction process is to assume that the average contact stress does not change during the penetration [16]. Equation (17) reduces to

$$\frac{1}{4}\rho_i h \pi (rv)^2 = \bar{\sigma} \int_0^{s_0} h(4rs - s^2)^{1/2} ds, \quad (20)$$

where $\bar{\sigma}$ is the impact crushing strength. As long as penetration s_0 is small relative to the floe radius, then it is given by

$$s_0 = \left(\frac{3\pi\rho_i}{16\bar{\sigma}} \right)^{2/3} v^{4/3} r. \quad (21)$$

The crushed ice volume for a single collision is simply

$$V = \frac{\rho_i h \pi}{4\bar{\sigma}} (rv)^2. \quad (22)$$

The constant stress assumption is reasonable if the thickness of the floe is not large relative to the horizontal contact length b and as long as larger scale fracture is suppressed. One advantage of the approach is that indices of crushing strength have been tested extensively at various loading rates and conditions. A disadvantage is that the rate of loading varies rapidly during impact and contact stresses can increase with increasing penetration and contact area. Crushed ice volumes were estimated as a function of crushing strength and are shown in Figure 6.

Simple solid and fluid relations were given above for the collision between floes. Other more comprehensive approaches are possible but, until a better understanding of crushing and fluid properties exists, the complexity is not warranted.

WAVE ATTENUATION AND FLOE SIZE REDUCTION

Since it is assumed that the floe collisions are completely inelastic we can use equation (13) to estimate the energy lost to the wave/ice system by floe impacts, and compare this to measured values of wave energy loss. The relation (12) for collision velocity was derived for an isolated floe pair with an optimal average floe separation of $2r$. Extrapolating to a floe field, we separate these floe pairs by a distance of 0.4 m such that no interference occurs between the pairs (see Figure 3). Using the example of a 0.1 Hz, 1 m amplitude wave, and 10 m diameter floes, the energy lost in a single impact is 11.0 J m^{-2} . Wadhams et al. [4] show that wave energy decay in MIZ's can be approximated by

$$E_x = E_0 \exp(-\alpha x), \quad (23)$$

where E_0 is the energy incident on the ice edge, E_x is the energy reaching a distance x into the ice covered region, and α is an energy attenuation coefficient dependent on floe size, percent coverage, ice thickness, wave amplitude and wave period. Measurements in the Labrador Ice Margin have shown α to be approximately $8 \times 10^{-4} \text{ m}^{-1}$ [17]. The incident wave energy ($\frac{1}{2}\rho g a^2$) is 5023 J m^{-2} . At a distance of 20 m into the ice cover, the wave energy is reduced to 4943 J m^{-2} — a reduction of 80 J m^{-2} . Therefore the

energy lost in the inelastic collisions is 14% of the total energy lost. This percentage is independent of wave amplitude. Since the input values used in this example are expected to give upper bounds on the impact energy losses, we expect that in a real ice cover this ratio would be even smaller, suggesting that collisions play a significant but not large role in wave energy attenuation. Some caution should be advised in this interpretation since these calculations are sensitive to changes in α .

The collision process does however influence floe size which is now shown for the conditions given above. Rewriting equation (23), the wave amplitude a at a distance x into the ice pack is

$$\left(\frac{a}{a_0}\right)^2 = \exp(-\alpha x), \quad (24)$$

where a_0 is the initial wave amplitude. If the floes collide once during every wave cycle and maintain their circular shape, then the rate at which the radius decreases is

$$\frac{dr}{dt} = -\frac{V}{4\pi hrT}, \quad (25)$$

where V is the volume of crushed ice per collision and T is the wave period. Note that the total volume of the floe pair was used in deriving (25). Substituting for crushed ice volume from the constant stress relation (22) and for collision velocity from (12), equation (25) can be rewritten

$$\frac{dr}{dt} = -\frac{\rho_i r^3 (\omega k)^2 a^2}{4\sigma T}, \quad (26)$$

where the constants ω and k are the wave number and the wave frequency. If the initial floe radius is r_0 , then the radius at any given time is obtained by integrating the last expression to yield

$$r = \left(\frac{1}{r_0^2} + \frac{\rho_i (\omega k a)^2}{2\sigma T} t \right)^{-1/2}. \quad (27)$$

Floe size reduction at any distance from the ice edge can be calculated from equations (24) and (27). The results for a constant stress σ of 1 MPa, an initial floe radius of 5 m and the incident wave conditions noted above are shown in Figure 7. A floe size reduction from 5 m to 4.25 m results in a crushed ice volume that fills the interstitial space between floes; further brash ice production could potentially influence the collision process. Wadhams et al. [4] have shown that α varies inversely with r , but this is not expected to be significant for this range of floe sizes.

CONCLUSIONS

A simple framework for floe decay and wave attenuation in marginal ice zones has been presented. The following conclusions were made from the collision of 10 m diameter, 1 m thick floes subjected to 1 m amplitude waves with a 10 s period. By assuming that floe collisions are inelastic and that floes collide directly with the maximum wave induced velocity, the collisions contribute less than 15 percent of the total wave attenuation in the ice pack. Collisions are important for determining the decrease in floe size. At the ice edge, a 10 percent floe diameter reduction would take approximately 4 hours.

The floe size distribution, the separation between floes and the eccentricity of the collisions are expected to influence large-scale wave-ice interactions. Further modelling of marginal ice zone collision processes should incorporate these factors, but must be accompanied by comprehensive field measurements of collision frequency, impact velocity and floe size reduction. The mechanical grinding of floes during contact may also be

significant. Simple assumptions have been made regarding ice properties by considering constant impact pressure and Newtonian fluid models. Experiments are necessary to address the damage and flow properties of sea ice at temperatures near to the melting point. A more comprehensive model of the ice collision process that includes ice damage and flow of crushed ice is currently being developed.

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